Differential Color Refraction Correction in the TOPP
RLS I/2006

The Differential Color Refraction (DCR) is the only offline calibration that needs to made in the Torino Observatory Parallax Program (TOPP). All the observations in the TOPP program are in the I filter so we expect the DCR to be small, here we estimate it's value and compare it to other determinations. The method used to calibrate the DCR follows that of Monet et al. (AJ, 103:638, 1992) which is basically to observe a number of fields over a range of hour angles (or more importantly varying zenith distances in the x coordinate) and compare the differential shifts of the field stars compared to the mean position as a function of color.

This method rests on the following reasoning:

1) That the components of refraction can be represented by the relations:

$R_x = \alpha A Z_x$ and $R_y = \alpha A Z_y$

where α is a product of a wavelength dependant quantity and the physical conditions (mainly temperature and pressure) of the air, and the Z's are the projected zenith distances in the tanget plane.

2) That the wavelength dependant quantity is related monotonically with the (V-I) color of the objects and that the physical conditions do not vary greatly over the course of the observing program then these relations simplify to:

$R_x = F Z_x$ and $R_y = F Z_y$

where F is a function of V-I which we determine empirically . We can simplify further by as summing that the shifts are relative to the field in meridian redefining the Z_y to be 0 in meridian.

See Monet 1992 for a more detailed discussion of the above reasoning.

In figure 1 we plot the variation of Z_x and Z_y with hour angle for three of our targets at the extremes and middle of our declination range. In the legend we return the values of the two zenith distances for hour angles of +1 and -1 the limit of our observations. We see that Z_x varies by a factor of 5 times the Z_y which indicates the components of relative refraction are largest in the X coordinate and it is in that axis we calibrate it.

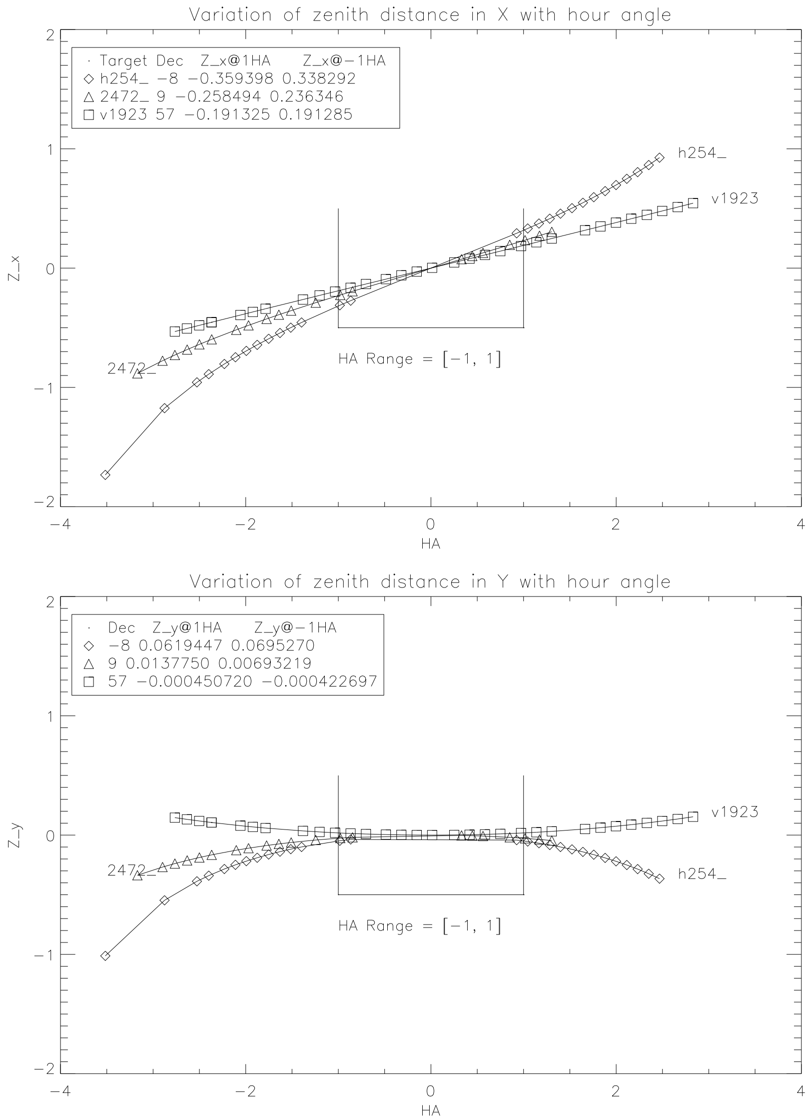

Figure 1: Variation of the zenith distance in the X and Y coordinates with hour angle for three example targets from the TOPP program. In the legend we list the values of the respective zenith distances at hour angles of -1 and 1, the limits of observtions in the TOPP.

Single Sequence Observations

In table 1 we list the statistics of the observations made for this calibration. A number of the sequences were dropped for various reasons:
1) Number stars in common too small (mdwr)
2) Range in Z_x too small (1821,l147,3536)
3) Noise in fit large (l243,1302,l334)
4) Range in $V - I$ too small (l347,3164,l360,l316)
The last three sequences (v1475,v1923,vh254) were also not used in the main calibration as they were for observations made in the V filter.

TABLE 1

Stars	Frames	min Z_x	max	min V-I	max	Target
3	12	0.242	1.385	2.274	4.410	mdwr
18	12	-0.140	0.648	0.385	1.694	1302
72	28	-1.733	0.926	0.567	3.877	h254
28	15	-0.971	-0.307	0.541	2.659	1547
105	34	-0.904	0.113	0.946	3.926	l474
15	8	0.037	0.285	0.672	1.307	l316
20	14	0.074	0.590	0.563	1.307	3164
14	30	-0.681	1.051	0.364	1.803	l360
10	21	-0.882	0.305	0.479	2.786	2472
104	8	-0.009	0.264	0.442	2.016	1821
43	15	-0.071	0.165	0.420	1.512	3536
17	19	-1.198	0.175	0.672	3.006	mrse
12	14	-1.075	0.078	0.452	1.471	l347
19	78	-0.688	0.205	0.342	2.719	l551
15	22	-0.073	0.723	0.740	5.027	2632
16	26	-0.531	0.448	0.371	3.083	l243
42	82	-0.114	0.625	0.465	2.688	h334
41	66	-0.084	0.707	0.465	2.688	s334
40	13	-0.866	0.000	0.465	2.688	l334
83	9	-0.057	0.151	1.011	5.127	l147
93	34	-0.282	0.539	1.011	2.933	v1475
43	32	-0.531	0.544	0.758	4.154	v1923
80	20	-0.318	0.632	0.246	3.877	vh254

In figure 2 we show the relative positions of the reddest star (LHS254) and a field star in the h254 sequence, the noise of the observations is an indication of our inherent difficulty, we are trying to measure a quantity that has a variation between the field and the reddest star of the order of our measuring precision (10-15mas). For each star we make this plot and determine a slope. From these slopes we attempt to find a correlation between difference per unit zenith distance (the slope) and the color. For the field h254 this correlation is shown in figure 3 for both the x coordinate and the y coordinate. The three lines correspond to different linear fits, LS-O is

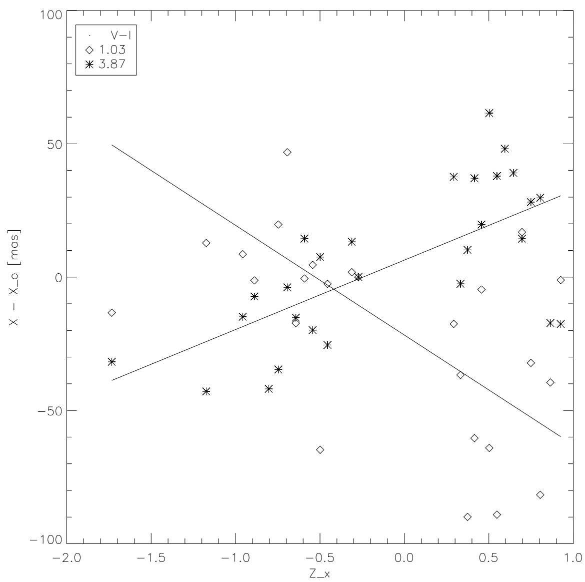

Figure 2: The relative shift of a medium color star and a red star for the sequence h254.

a least squares with 5 sigma outliers removed, LS is a least squares with all the data and MAD is a minimization of the absolute deviations.

To examine the possibilities of correlations with the combining of frames and the DCR effect we tried 4 different sets of stars to tie frames together:
0) All stars in common
1) 12 stars in common with lowest residuals after pre-fit
2) 12 stars in common with colors closest to the mean color
3) Stars without V-I colors

In table 2 we show the parameters of the linear fits using the LS-O method for the 4 combinations of tie stars in the h254 field.

TABLE 2

Method	AX	AXerr	BX	BXerr	AY	AYerr	BY	BYerr
0	-7.52	6.14	5.96	5.33	14.43	33.55	-10.16	29.27
1	-3.04	7.23	1.70	6.37	41.50	34.96	-18.18	30.67
2	-8.64	6.26	6.92	5.48	17.44	25.49	-0.45	22.18
3	-1.34	7.22	12.94	6.26	-1.96	23.34	3.45	19.91

where the parameters are from the relation:
$X - X_o/Z_x = AX + BX * (V - I)$ and
$Y - Y_o/Z_y = AY + BY * (V - I)$

The parameter that is our best measure of the DCR is BX, the intersect AX is relative to the mean color of the field and will change if the stars

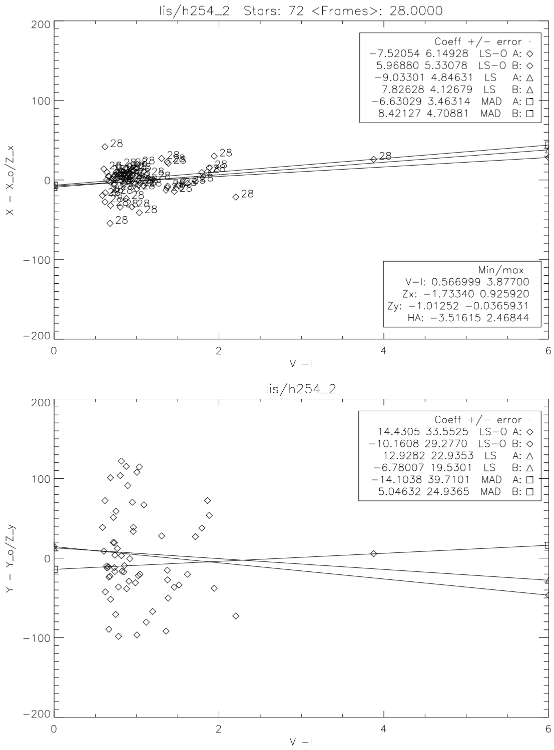

Figure 3: The slopes of the relative shifts for all stars in the field of h254 (LHS254).

used in a field are varied. For the first three methods in h254 BX cannot be distinguished from 0. Since we do not expect the correlations to be high as there are at least 12 stars in common and only two parameters in each coordinate for the next step we use the results from method 0 though the results form the other methods were also examined.

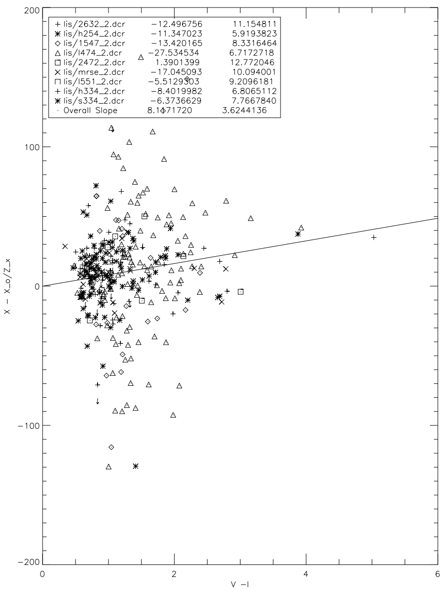

Figure 4: The slopes of the relative shifts for all stars in all fields with variable zero point shifts for each field.

Tying the sequences together

As each sequence is measured relative to itself the intersects found are all relative a better estimate of the true slope can be found by combining the data from all the sequences. After examining the sequences individually and removing those listed above we are left with 9: h254, 1547, l474, 2472, mrse, l551, 2632, h334 and s334. The individual slopes and colors for all the stars were combined into a single system with 10 parameters, 9 for the zero points which are relative to the color distribution of the field and 1 for the correlation of the DCR with V-I. Combining fields provides us with more red stars to fix the fit as in the individual cases only one star, the target star, has a color greater than 2.5 with the combined fields we have 6 stars.

In the legend we list the various zero points and the final slope of 8.1 +/- 3.6 this is marginally significant hence in the TOPP program we apply the correction in X:

$Xcorrected = Xobserved - 8.1 * V - I * Z_x$

also similarly in Y.

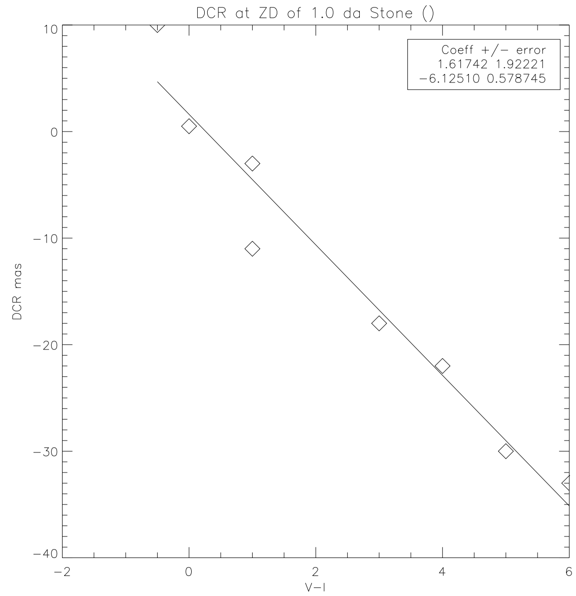

Figure 5: The DCR calibration as provided by Stone (2002) for the DCR in I at a Tan(ZD) of 1.

Comparison to Other Calibrations

For the target LHS254 with a V-I of 3.87 and a declination of -8 the Z_x at an HA of 1. is -0.36 so the correction for an observation at this HA is 11.2mas.

In Stone (**2002**, PASP 141:1070) a relation for the absolute DCR shift at flagstaff and color is found for various colors, at a ZD of 45 degrees, e.g. a tan(ZD) of 1.0. The relation for V-I colors was provided directly from Stone via a private communication and is shown in figure 4. A linear fit provides the equation: $DCR = 1.6 - 6.1(V - I)$

For the target LHS254 this provides a DCR shift of -22mas @ a TAN(ZD) of 1. Equation 3 from Stone allows us to translate this value to the corresponding value at Torino:

$DCR' = [(t + 273.15) * P' * tan(ZD')]DCR/[(t' + 273.15) * P * tan(ZD)]$

where t=10.9, P=581.8 and tan(ZD)=1. for the Flagstaff program. On the night of the sequence h254 07/01/1995 t'=-1 and P=710, so for a zenith distance in x of -0.36 then:

$DCR' = ((10.9+273.15)*710.*(-.36))*(-22)/((-1.+273.15)*581.8*1.)$

This provides a DCR of 10.1 mas in good agreement with the 11.2mas found empirically.

The work of Jao (AJ 129:1954 **2005**) who find $DCR = 0.7(V - I)$ cannot as it stands be reconciled with these results.

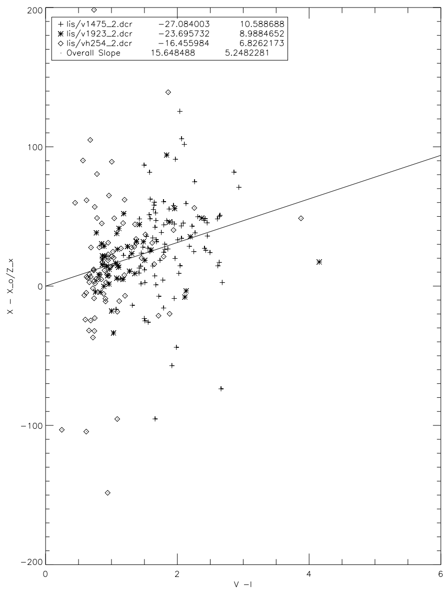

Figure 6: The DCR calibration for the V band, analog to figure 4.

DCR in the V band

Another test is to measure the DCR in the V band. This should be larger than that in I but as pointed out in Jao (2005) not as large as that in R. In the list of sequences there are 3 V band sequences: v1475 v1923 and vh254, in figure 6 we plot the combined slopes for these three sequences the analog to figure 4. The DCR in V is nominally twice as large, in Stone he finds a DCR in V 3-4 times larger which is not inconsistent with this result considering the errors on the various parameters.